A New Choice in Multi-Period Investment Performance Attribution: Effective Return versus Geometric Smoothing

An ongoing challenge in multi-period performance attribution is getting numbers to add that do not add naturally. Specifically, the benchmark return plus the sum of attributed effects (like selection and allocation) should equal the reported return. In [Surz, 2010], I introduced a new method called "effective return" that produces the desired relationship by solving for component returns whose weighted sum equals the known rate of return. In this sequel to that article, I compare and contrast effective return to the geometric smoothing methods that have been used previously. Effective return is an alternative to geometric smoothing. Both approaches are kluges, so choose your favorite kluge. The appeal of effective return is that the kluge actually tells us something about the effectiveness of portfolio changes through time.

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Attribution aficionados are very familiar with the sophisticated mathematics that force multi-period attribution calculations to add up. These "smoothing algorithms" conform multi-period attribution calculations to single-period structures. Specifically, we want the following relationship to hold:

ATTRIBUTION ARITHMETIC REQUIREMENT

Benchmark Return + Selection + Allocation = Fund Return

This seems straightforward enough on its surface and would not be a challenge if investment returns were additive, but they are not. Investment returns compound (link geometrically) so some form of plug, or kluge, that must be applied to create the desired arithmetic. Several such smoothing algorithms have been devised and are in the public domain, plus others have been homegrown, so they are proprietary. Algorithms in the public domain include:

- [GRAP, 1997]
- [Cariño, 1999]
- [Menchero, 2000]
- [Davies & Laker, 2001]
- [Frongello, 2002]

The problem with all of these approaches is that the researcher must trust a black box operating in the background because you cannot see the relationships at the sector level. Attribution effects at the sector level bear no verifiable relationship to sector performance and commitment. This is because the behind-the-scenes manipulations are quite complex. Table 1 is an example of the disconnects. Columns A through D are the building blocks for single-period attribution, but as you will see there is no way to verify the attribution contributions using these columns. These columns are:

- Columns A and C are the average allocations through time to the fund and the benchmark, respectively
- Columns B and D are the geometrically linked compound returns of the portfolio and the benchmark, respectively

Please note that the attribution measures shown in columns E and F cannot be readily reconciled with columns A through D, as they can in single-period attribution. But the "good" news is that we can solve for

Table 1: Multi-period Attribution with Geometric Smoothing

Note that Columns A-D cannot be related to attribution in E and F.

	Portfolio		Benchmark		Skill		
	A	B	C	D	E	F	
Market Segment	Weight	Return	Weight	Return	Allocation	Selection	Value Added
Consumer Staples	6.95	-31.64	3.67	36.29	4.55	-12.61	-8.07
Consumer Discretionary	14.12	-28.74	3.62	19.74	13.62	-21.45	-7.83
Health Care	5.73	130.72	1.26	33.67	2.22	3.29	5.50
Materials	10.74	37.28	14.95	-4.32	0.87	4.84	5.71
Information Technology	12.95	179.68	16.48	14.48	-0.02	15.62	15.61
Energy	11.69	-9.74	16.27	-18.01	1.80	-5.00	-3.20
Industrial	1.84	-0.14	5.28	-5.97	-0.20	-0.14	-0.34
Telecom-Utilities	18.21	-21.54	15.92	-15.78	-0.59	0.76	0.17
Finance	17.78	-18.13	22.50	7.27	-0.60	-0.85	-1.44
Cash	0.00	0.00	0.05	0.85	0.00	0.00	0.00
Total	100.00		100.00		21.65	-15.54	6.11
Returns		6.52		bm = 0.39			

the kluge that is created by geometric smoothing, as shown in Table 2 below. Table 2 begins with "Raw" attribution measures that use single-period calculations applied to multi-period returns and average allocations, in columns A-D. We know these won't generate the desired arithmetic, so the columns labeled "kluge" show the difference between the "Raw" calculation and the smoothed results. As you can see, smoothing adds 16.88% to the raw Allocation measure and subtracts 30.91% from the raw Selection measure. This is an interesting exercise, but there is no way to interpret these kluges; they tell us nothing of value.

In this article, I add yet another approach to the discussion, and it's about time since not much has changed in the past decade. I call it "effective return." In a nutshell, effective return is the return that a stock or portfolio segment (sector, style, country, etc.) would need to earn to produce the known actual cumulative portfolio return. In [Surz, 2010] I describe one way to calculate effective return and speculate that alternative methods could be devised in the future. Here is the process I use:

- Step 1: Compute the preliminary effective return as the commitment (or allocation) weighted average of the security or portfolio segment across periods that span the report dates. This is similar to a money-weighted return in that it is an allocation-weighted return.
- Step 2: Estimate the preliminary impact of each security or segment by calculating its average alloca-

Table 2: Backing Out the Kluges in Geometric Smoothing

	Allocation			Selection		
	Raw	Kluge	Final	Raw	Kluge	Final
	(D-bm)*(A-C)		E	A*(B-D)		F
Market Segment						
Consumer Staples	1.18	3.37	4.55	-4.72	-7.89	-12.61
Consumer Discretionary	2.03	11.59	13.62	-6.85	-14.61	-21.45
Health Care	1.49	0.73	2.22	5.56	-2.28	3.29
Materials	0.20	0.67	0.87	4.47	0.37	4.84
Information Technology	-0.50	0.48	-0.02	21.39	-5.77	15.62
Energy	0.84	0.96	1.80	0.97	-5.97	-5.00
Industrial	0.22	-0.42	-0.20	0.11	-0.25	-0.14
Telecom-Utilities	-0.37	-0.22	-0.59	-1.05	1.81	0.76
Finance	-0.32	-0.28	-0.60	-4.52	3.67	-0.85
Cash	0.00	0.00	0.00	0.00	0.00	0.00
Total	4.76	16.88	21.65	15.37	-30.91	-15.54

tion across periods multiplied by its preliminary effective return.

- Step 3: These preliminary impacts are added up to provide a "trial return" on the portfolio. The trial return usually does not capture all of the joint compounding and timing effects, so an adjustment factor is required to true up the effective returns. The preliminary effective returns are rescaled by spreading the difference between the actual return and the trial return. The difference is pro rata allocated to each segment based on the absolute value of its preliminary impact. The result is that the sum of average allocation weighted effective returns equal the actual returns for the portfolio and benchmark, respectively.

Effective return causes all the multi-period attribution components to add in exactly the same way that they do in a single-period attribution, so all of the disconnects go away. It also creates a measure that better captures the decisions of the investment manager. In other words, effective return solves the multi-period problem by tying it back to the performances of the fund and the benchmark. We force the allocation-weighted sum of component returns to add to the known multi-period returns. This form of kluge comes with several benefits.

BENEFITS OF EFFECTIVE RETURN MULTI-PERIOD ATTRIBUTION

- Clear presentation of the kluge
- Interpret its effect as caused by compounding and changes in allocation
- Easy to see and understand
- "Raw" attribution contributions can be viewed as selection and allocation effects in the absence of compounding and reallocations, so the kluge captures and reports these compounding and timing effects.

Table 3 presents the same analysis as above, but uses effective return. Please note that the Raw attribution contributions in columns E and F are identical to Table 2, so the major difference is teasing out and explaining the kluges.

Column G shows a new measure designated as "Timing." The more appropriate description is "the combined effects of compounding and changing allocations." It is a kluge, but a kluge with information, as can be seen from the formula used to calculate it. Here are the components of the formula:

COMPONENTS OF "TIMING" (AKA THE COMBINED EFFECTS OF COMPOUNDING AND CHANGING ALLOCATIONS)

B'-B is the excess or shortfall of the effective return relative to the raw linked return. It is positive if sector allocation changes through time added value, or negative if they subtracted value.

D'-D is the excess or shortfall for the benchmark. Yes, benchmarks do have changing allocations to segments over time.

A(B'-B) –C(D'-D) is the allocation-weighted value add or subtracted on the portfolio, and we subtract from that the same measure as calculated on the benchmark.

Table 3: Same Attribution Analysis Using Effective Return

	Portfolio			Benchmark			Skill			
	A	B	B'	C	D	D'	E (Raw)	F (Raw)	G	
							(D-bm)*(A-C)	A*(B-D)	A(B'-B)-C(D'-D)	Value
Market Segment	Weight	Return	Ef Ret	Weight	Return	Ef Ret	Allocation	Selection	"Timing"	Added
Consumer Staples	6.95	-31.64	-42.49	3.67	36.29	138.98	1.18	-4.72	-4.52	-8.07
Consumer Discretion	14.12	-28.74	-21.82	3.62	19.74	130.12	2.03	-6.85	-3.02	-7.83
Health Care	5.73	130.72	107.31	1.26	33.67	49.97	1.49	5.56	-1.55	5.50
Materials	10.74	37.28	24.80	14.95	-4.32	-20.29	0.20	4.47	1.05	5.71
Information Tech	12.95	179.68	121.56	16.48	14.48	0.91	-0.50	21.39	-5.29	15.61
Energy	11.69	-9.74	-81.73	16.27	-18.01	-38.96	0.84	0.97	-5.01	-3.20
Industrial	1.84	-0.14	-1.67	5.28	-5.97	6.14	0.22	0.11	-0.67	-0.34
Telecom-Utilities	18.21	-21.54	-21.32	15.92	-15.78	-25.52	-0.37	-1.05	1.59	0.17
Finance	17.78	-18.13	8.26	22.50	7.27	13.03	-0.32	-4.52	3.40	-1.44
Cash	0.00	0.00	0.00	0.05	0.85	0.59	0.00	0.00	0.00	0.00
Total	100.00		6.52	100.00		bm = 0.39	4.76	15.37	-14.02	6.11

"Timing" is the net (of benchmark) value added or subtracted by changes in allocation through time.

In the example in Table 3, the timing measure implies that shifts in allocations across sectors subtracted 14.02%, with the biggest hits coming from InfoTech and Energy. This is an insight that simply doesn't exist with smoothing techniques.

Here are the main differences between geometric smoothing in Table 2 and effective return in Table 3:

Returns on the portfolio (6.52%) and the benchmark (0.39%) are in fact the allocation-weighted sum of effective returns, as shown in Table 3. Columns B' and D', when multiplied by columns A and C, respectively, sum to the known returns shown at the bottom of B' and D'. Smoothing algorithms do not provide this building block.

All three attribution measures in Table 3, shown in columns E through G, can be quickly and easily verified using the formulas shown at the tops of these columns. Columns E and F use standard single-period formulas, while the new column G provides an informative new measure that causes everything to add as desired. As stated in the introduction, the formulas used in smoothing algorithms are too complex to allow simple eyeball verification.

Simple comparisons of raw returns to effective returns, comparing column B to B', reveal another dimension in performance attribution – the effects of changes.

THE JOURNEY OR PATH OF MULTI-PERIOD ATTRIBUTION

Most researchers want to know the intermediate attribution results through time, as depicted in the exhibit. In this example, sector allocation was not a major contributor, and stock selection went through four distinct phases:

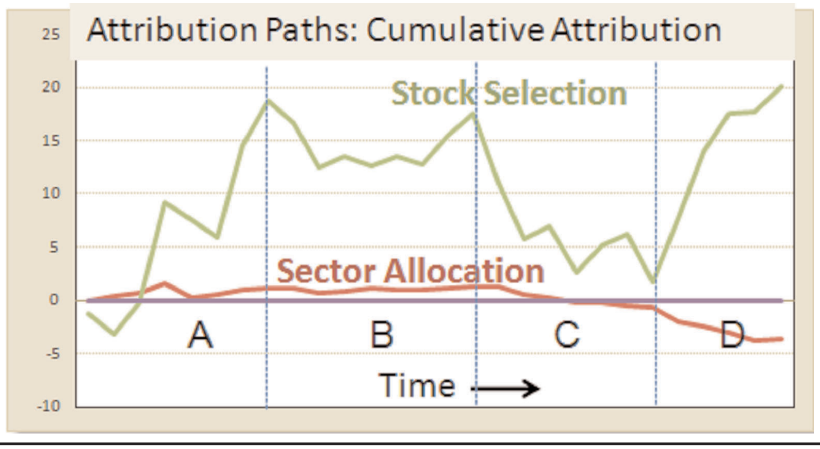
- Stock selection was very good in periods A and D
- It was flat peak-to-peak in period B
- And stock selection was poor in period C

As a result, the analyst may choose to run attribution reports on each of these subperiods individually to gain further insights into the manager's decisions. Why was period C poor while periods A and D were good?

Here, again, there is a choice. Intermediate attribution effects can be calculated by stringing together individual single-period attribution effects and smoothing them through time. In other words, stock selection and sector allocation in the exhibit could be calculated for each single period in the analysis range, and these individual single period results could be tied together in some fashion say, for example, using geometric smoothing.

The alternative is to bring attribution components, namely returns and allocations, forward through time and calculate a sequence of multi-period attribution results. This second approach is best suited for effective return attribution. In single-period attributions, there is no difference between segment returns and effective re-

Figure 1



turns; they are one in the same. In other words, the effective return approach performs a one-period analysis, followed by a two-period analysis, and then a three-period, and so on until the entire report range is covered.

CONCLUSION

You have new choices: effective return or geometric smoothing. Let the discussions begin anew. Discussions about smoothing techniques were commonplace in the mid-2000s, but nothing new had come along, until now.

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