An ongoing challenge in performance attribution is getting numbers to add that
do not add naturally. Specifically, the benchmark return plus the sum of
attributed effects (like selection and allocation) should equal the reported
return. This is not a problem for a single-period analysis because the
benchmark return and the attribution effects add up to the portfolio’s total
return by definition, but for multiperiod analyses, returns compound (grow
gonomically) rather than add, so there is a disconnect. As a result, several
“smoothing” techniques have been developed over the years, and debate
continues about which one is better. In this article, I add yet another approach
to the debate called “effective return.” In a nutshell, effective return is the
return that a stock or portfolio segment (sector, style, country, etc.) would need
to earn to produce the known actual cumulative portfolio return. Effective
return causes all the multiperiod attribution components to add in exactly the
same way that they do in a single-period attribution, so all the disconnects go
away. It also creates a measure that better captures the decisions of the
investment manager. There are several ways to solve for effective return. I
provide one approach here. But before I get into the details, I will start with a
description of the objective.

**The Big Picture**

The objective is to provide reports for cumulative periods that adhere to the
same logic as single-period attribution. Specifically, the goal is to have the
formulas used for “allocation” and “selection” actually work over multiple periods. These formulas are usually shown for single-period attribution and then removed from multiperiod attribution reports because they do not work. **Exhibit 1** contrasts traditional multiperiod attribution to the new approach that uses effective return. Please note that in traditional attribution, disconnects are caused by the simple fact that returns compound rather than add:

1. Total returns are not the commitment-weighted sum of component returns. Fixing this disconnect is, in fact, the magic in effective return attribution.
2. As a direct result of the first disconnect, attribution components do not tie to anything on the report because the smoothing algorithms that force geometric sums to arithmetically add take place in the background in a complex black box.

**Exhibit 1. Traditional Multiperiod Attribution vs. Effective Return Attribution**

<table>
<thead>
<tr>
<th>Traditional Geometric Attribution</th>
<th>Effective Return Attribution</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Market Segment</strong></td>
<td><strong>Portfolio</strong></td>
</tr>
<tr>
<td>Consumer Staples</td>
<td>12.21</td>
</tr>
<tr>
<td>Cons Discretionary</td>
<td>8.92</td>
</tr>
<tr>
<td>Health Care</td>
<td>14.11</td>
</tr>
<tr>
<td>Materials</td>
<td>3.31</td>
</tr>
<tr>
<td>Technology</td>
<td>17.95</td>
</tr>
<tr>
<td>Energy</td>
<td>12.55</td>
</tr>
<tr>
<td>Industrial</td>
<td>10.31</td>
</tr>
<tr>
<td>Telecom Utilities</td>
<td>7.55</td>
</tr>
<tr>
<td>Finance</td>
<td>13.10</td>
</tr>
<tr>
<td>Cash</td>
<td>0.00</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>100.00</strong></td>
</tr>
<tr>
<td><strong>Portfolio</strong></td>
<td><strong>100.00</strong></td>
</tr>
<tr>
<td><strong>Skill</strong></td>
<td></td>
</tr>
</tbody>
</table>

Exhibit 1 clearly shows that the weights (allocations) of the two approaches are

**Total Returns are NOT sum of Weight X Return**

**Attribution components do not tie to anything on the page**

**Total Returns are sum of Weight X Return**

\[
\sum (A \times B) \& \sum (C \times D)
\]

**Attribution Calculated as Shown at top Of Column**
identical. Only the component returns are different, and it is this difference that ties the effective return attribution components together because the effective return approach produces total returns that are the allocation-weighted sum of the component returns.

Notice that the standard attribution formulas shown above the allocation and selection columns on the right side of Exhibit 1 actually work. This is a live report for a 12-month period that links monthly attribution results and solves for effective return. Also, please note the differences between the traditional and effective approaches in the component returns. Traditional attribution shows the return on a sector held in isolation with a unit weight—the growth of a dollar held at the beginning of the period. In contrast, the effective return incorporates changing allocations to the sector through time, thus capturing management decisions. Even the benchmark has changing allocations through time because of market movements and rebalancing, so differences are evident in component returns for the benchmark as well as the portfolio: Columns B and D are different between traditional and effective.

Something else to note is that nothing about the attribution components in the traditional approach (the left side of Exhibit 1) can be confirmed—other than the fact that the attribution columns actually add to the totals shown. The black box smoothing algorithms, which typically use ratios of logarithmic sums, are too complex to show on traditional attribution reports, so the analyst must make a leap of faith.

The interesting fact is that both approaches usually provide approximately the same total attribution results, but the effective return reveals more about the manager’s skill and decision making. In particular, effective return captures the effects of the manager’s allocation decisions through time; it is a decision-weighted performance result. In contrast, geometric smoothing uses a constant weighting to market segments within the portfolio through time, which is rarely the case in reality.
The Challenge

The challenge is to have the sum of “impacts” equal the cumulative return of the portfolio, where impact is defined as allocation times effective return. It seems natural to calculate allocation as the average percentage commitment to a stock or segment, so the challenge is teasing out an effective return. The idea is that the allocation-weighted sum of the effective returns is, in fact, the reported return. At PPCA, we have developed a two-step algorithm to solve for effective returns:

1. Calculate a preliminary, trial, effective return as the commitment-weighted average return on the stock or segment, which produces a preliminary impact, calculated as average allocation times trial effective return. This first step captures individual stock effects. We have found that it works best if the return on the actual portfolio is proxied for the stock’s return during periods when the stock is not held. The view is that the manager has the option of holding either the stock or the portfolio without the stock.

2. Calculate the difference between the actual return and the sum of allocation-weighted trial impacts, and pro rata (using trial impacts) allocate this delta across all stocks or segments. This second step captures interaction effects, especially over lengthy cumulative periods.

This algorithm produces effective returns that are near the known stock or segment returns if allocations are constant through time (i.e., positions are constantly rebalanced). Deviations from constant rebalancing reflect presumed trading decisions, including buy and hold (the decision is not to trade).

Some real life examples provide further insight. In 2009, Proctor and Gamble (P&G) returned 1.23 percent, but its effective return in the S&P500 Index was –5.19 percent. At first blush, this would seem to be an error because the S&P 500 is mostly a buy-and-hold portfolio, and P&G was in the S&P 500 for the
entire year. Figure 1 reveals what happened. P&G started the year with losses in excess of 10 percent, reducing its weight in the S&P 500, so it had its lowest allocations when returns subsequently recovered. Note also that the effective return for P&G is unique to each portfolio, reflecting not only trading in this company but also its relative allocation within the portfolio.

Figure 1. Effective Return Analysis, 31 December 2008 to 31 December 2009

In many cases, especially over long time periods, effective return is substantially different from the simple return on a stock or portfolio segment because effective return reflects the allocation decisions that have been made through time. Conventional return will equal effective return if and only if the allocation to a stock or segment is held constant (e.g., a constant rebalance to a 5 percent allocation). Figure 2 from an effective return report illustrates this point. The published return on the energy sector over the report period is +50 percent, yet the effective return is −50 percent.
This difference would appear to be an anomaly. But the difference is explained by a disadvantageous increase in the allocation to the energy sector just prior to significant losses. As one can see in Figure 3, allocation was more than doubled in July 2007, just prior to significant losses in the sector, and then increased again in July 2008.
Effective return is unique to each portfolio because it incorporates the effects on wealth of changing allocations.

Conclusion

Effective return produces the desired mathematics for attribution components to add over cumulative periods. It also provides an additional insight when contrasted to the known reported return over the evaluation period, representing allocation success or failure at the individual stock level, an inference that is important to evaluators and investment managers alike. The devil is in the details, but the benefits are in the results. Even if you have not followed the algorithm just described, you can still see how this approach can pinpoint trading successes and failures in a way that has not been provided before.