The Truth about Diversification by the Numbers

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"Ninety-five percent of the benefits of diversification are captured with a 30-stock portfolio." This is a common belief held by virtually all investment professionals. It's based on research conducted by professors Lawrence Fisher and James H. Lorie (F&L) on NYSE-traded stocks during 1926-1965.¹ In this article we clarify F&L's work, offer a couple of alternative approaches, and update the analysis to incorporate January 1, 1986 through June 30, 1999.

F&L measured the percent of all *possible* reduction in *dispersion* achieved by portfolios of various sizes on *average*. The reductions were measured relative to the dispersion of a one-stock portfolio. In this context, "all possible reduction" is the denominator of the ratio and is the difference between the dispersion of a one-stock portfolio and the dispersion of a portfolio comprising all NYSE stocks. The numerator of the ratio is the difference in dispersion between a one-stock portfolio and that of an "N"-stock portfolio were "N" is the number of stocks. Their results are summarized as follows:

Table 1

Percent of possible reduction in dispersion achieved through increasing the number of stocks in the portfolio.

<u>1</u>	<u>2</u>	<u>8</u>	<u>16</u>	<u>32</u>	<u>128</u>	All <u>Market</u>
0	41	82	90	95	99	100

Hence it is commonly said that 90% of diversification is achieved with a 16-stock portfolio, and 95% is achieved with a 32-stock portfolio. We believe this is a misinterpretation of the F&L results, for reasons presented in the following.

F&L measure the reduction in *total* volatility, which includes both diversifiable, or specific, and non-diversifiable, or market, risk. Modern Portfolio Theory (MPT) postulates that only market risk is rewarded in the aggregate, so specific risk is to be avoided.

The reduction in specific risk is the benefit of diversification, and the basis for measures of diversification such as R-squared and tracking error. R-squared measures the percent of variance that is explained by the market, and is hence undiversifiable risk. Tracking error measures specific, or diversifiable, risk as the standard deviation of returns away from the market.

R-squared and tracking error are the measures that should be used to determine improvement in diversification since they are measures of diversification. Improvement in overall risk is interesting, but does not support statements about diversification. Some real examples will illustrate the point. We have repeated the F&L analysis using Portfolio Opportunity Distributions $(PODs)^2$ for the period 1/1/86 to 6/30/99. PODs create all possible portfolios of a given size that could be held from stocks in the Compustat database, so the market in this analysis is broader than just the NYSE and includes NASD-traded securities. Table 2.a shows an F&L dispersion measure (standard deviation) and the two diversification measures. Table 2.b shows the percent of possible reduction derived from table 2.a:

1 abic 2.a. Risk and Diversification measures from $1/1/00 = 0/30/33$ for portionos of various si	Table 2.a:	Risk and Diversification	Measures from 1/1/86	5 – 6/30/99 for	portfolios of	various size
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	<u>1</u>	<u>15</u>	<u>30</u>	<u>60</u>	All <u>Market</u>
Standard Deviation	45%	16.5%	15.4%	15.2%	14.5%
R ²	0	.76	.86	.88	1.0
Tracking Error	45	8.1	6.2	5.3	0

Table 2.b: Percent of possible reduction

	<u>1</u>	<u>15</u>	<u>30</u>	<u>60</u>	All <u>Market</u>
Standard Deviation	0%	93%	97%	98%	100%
\mathbf{R}^2	0	76	86	88	100
Tracking Error	0	82	86	88	100

As can be seen from Table 2.b we get results that are very similar to F&L for reductions in dispersion, or standard deviation, but the results for improvements in diversification are much less than previously thought. We now see that a 15-stock portfolio gets 76% of available diversification versus the F&L 93%; this improves somewhat to 82% if tracking error is used as the diversification measure, but note that the tracking error is still a formidable 8.1% per annum. Even at 60 stocks we still have less than 90% of the available diversification whereas F&L would suggest virtually full diversification at this level.

In addition to correcting the misunderstandings about the F&L work, we want to extend our analyses beyond just the average fund to encompass the full range of results. Table 3 shows the ranges of dispersion and diversification for various size portfolios. As can be seen in the table some 15-stock portfolios have *less* dispersion than the market, but none come close to the diversification of the market. Seen this way it seems so obvious that reductions in dispersion do not equate to improvements in diversification, but the old interpretation of the F&L work will probably live on.

We also want to acknowledge that most managers attempt to diversify beyond a randomly chosen portfolio. Table 4 shows how tracking error can be reduced with two different techniques – optimization and holding the largest names. As can be seen computer optimization can significantly reduce diversifiable risk, but a less sophisticated approach of just holding the largest names can go a long way toward controlling tracking error.

Summary

The relationship between number of stocks held in a portfolio and diversification has been clarified. 15-stock portfolios, on average, achieve only 75-80% of available diversification, not the 90%-plus previously believed. Even 60-stock portfolios achieve less than 90% of full diversification.

Conscious efforts to diversify can improve these figures, but even optimizations won't achieve the diversification levels that were previously believed to be reached with simple random portfolios.

The implications of these findings for both the portfolio manager and the investor are significant. The portfolio manager can no longer rely on a simple rule of thumb to decide on the number of stocks to include in the portfolio. Diversification is more complex than the old "30-stock" saw had suggested. Similarly, investors should be less sanguine in the achievement of their diversification objectives if this confidence has been based on a count of the names held in their portfolio.

References

- Fisher, Lawrence and Lorie, James H. "Some Studies of Variability of Returns on Investments in Common Stocks." *Journal of Business Vol 43, No. 2, April 1970.*

Graphs may follow, but following tables are in article :

		Standard Deviation				
	15 Stocks	30 Stocks	60 Stocks	<u>Market</u>		
5	19.2	17.5	17.2			
25	17.5	16.1	16.0			
50	16.5	15.4	15.2	14.5		
75	15.5	14.7	14.5			
95	14.3	13.9	13.6			
		<u>R-Squ</u>	ared			
5	.86	.91	.94			
25	.84	.89	.91			
50	.76	.86	.88	1.0		
75	.71	.84	.86			
95	.63	.76	.79			

Table 3 : Ranges of Dispersion and Diversification

Table 4 : Reducing Tracking Error

		Tracking Error		
	15 Stocks	<u>30 Stocks</u>	<u>60 Stocks</u>	
Random	8.1	6.2	5.3	
Largest	7.5	5.2	4.1	
Optimized	5.4	4.2	3.5	